

齿轮弯曲疲劳可靠度预测的计算方法

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【摘要】 用一个新的基本观点—把齿轮弯曲疲劳失效处理成随机事件,以这个随机事件为研究对象,用齿轮弯曲疲劳的累积损伤强度 K_w 和总疲劳损伤量 D_w 来描述弯曲疲劳失效随机事件,得出用概率描述的弯曲疲劳的疲劳累积损伤模型。用这个计算模型,提出了计算齿轮弯曲疲劳可靠度的方法。

主题词: 齿轮 弯曲疲劳 疲劳损伤 计算方法 可靠性

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1 前言

齿轮弯曲疲劳失效是齿轮常见的疲劳失效形式之一,齿轮轮齿折断或失效会产生严重的事故。在现有的各种手册中,齿轮弯曲疲劳极限是用区域图表示的,这说明弯曲疲劳极限是个随机变量,对齿轮的弯曲疲劳问题应该采用随机变量的观点来处理。文献[1]采用应力—强度干涉理论来计算齿轮的弯曲疲劳可靠度。这种方法可用来处理等幅弯曲疲劳载荷的可靠度计算问题,但用文献[1]所述的方法处理多级等幅或随机变幅弯曲疲劳的可靠度计算问题则较困难。本文从一个新的基本观点出发,把齿轮弯曲疲劳失效处理成随机事件,提出了计算多级等幅或随机变幅载荷作用时齿轮弯曲疲劳可靠度的方法。

2 弯曲疲劳累积损伤的概率模型

齿轮在每次啮合过程中都产生一定的疲劳损伤,当疲劳损伤累积到一定程度时,齿轮便产生弯曲疲劳失效。疲劳损伤是个抽象的概念,一般都采用间接度量法来描述疲劳损伤量。定义^[2]:在一次弯曲应力循环中,齿轮产生弯曲疲劳的疲劳损伤为 δ

$$\delta = (\sigma_F)^m \quad (1)$$

σ_F 为齿轮某个弯曲应力循环的应力幅,是个随机变量(N/mm^2), m 为齿轮弯曲疲劳S-N曲线的指数。 m 本身也是个随机变量,但为了突出主要因素及简化运算,把 m 处理成确定性的量。

齿轮弯曲疲劳失效是一个随机事件。为了研究这个随机事件,必须用随机变量来描述。根据齿轮弯曲疲劳的一次循环疲劳损伤量 δ 的定义,可用下面二个随机变量 K_w 和 D_w 来描述齿轮弯曲疲劳失效随机事件。

$$K_w = N_T (\sigma_{FT})^m \quad (2)$$

σ_{FT} 为齿轮疲劳试验时弯曲应力循环的应力幅,即试验时的应力水平(N/mm^2); N_T 为齿轮试验

时,在 σ_{FT} 作用下齿轮产生弯曲疲劳失效的应力循环次数。

称 K_w 为齿轮弯曲疲劳的疲劳累积损伤强度,它反映齿轮抵抗弯曲疲劳失效的能力。 K_w 是个随机变量。当获得 n 个试验数据时,用式(2)计算出 K_w 的 n 个样本值,对这 n 个样本值进行统计分析,便能统计出 K_w 的分布规律和分布参数。

$$D_w = N \int (\sigma_F)^m f(\sigma_F) d\sigma_F \quad (3)$$

N 为总的应力循环次数(或寿命), $f(\sigma_F)$ 为弯曲疲劳应力水平 σ_F 的概率密度函数。

称 D_w 为齿轮弯曲疲劳的总疲劳损伤量,它反映齿轮在啮合过程中,在某一定的循环次数(寿命)内承受弯曲疲劳应力作用时产生的疲劳损伤的累积总和。 D_w 是随机变量,它由应力谱(载荷谱)及具体的计算类型而定。

当 D_w 小于 K_w 时,便不产生齿轮弯曲疲劳失效这个随机事件。因而可靠度 R 为

$$R = P(K_w > D_w) \quad (4)$$

式(4)是用随机变量 K_w 和 D_w 描述的,齿轮弯曲疲劳失效随机事件的概率表达式,也即是用来计算齿轮弯曲疲劳可靠度的弯曲疲劳累积损伤概率模型。

3 齿轮弯曲疲劳的疲劳累积损伤强度 K_w

在工程应用中,常常难于获得对应计算问题的齿轮弯曲疲劳的试验数据。下面讨论如何利用现在人们常用的齿轮手册^[3]来计算 K_w 。

工作齿轮齿面的弯曲疲劳极限 σ_{FP} 为^[3]

$$\sigma_{FP} = \sigma_{FLim} Y_{ST} Y_{\delta relT} Y_{\delta relT} Y_{R relT} Y_X \quad (5)$$

Y_{ST} 、 $Y_{\delta relT}$ 、 $Y_{R relT}$ 和 Y_X 为正态分布的随机变量,它们的均值按国标方法计算或查齿轮手册,其变异系数为^[1]

$$C_{YST} = 0.033, \quad C_{Y\delta relT} = 0.03, \quad C_{YR relT} = 0.033 \\ C_{YX} = 0.02 \quad (m_n > 5\text{mm}), \quad C_{YX} = 0 \quad (\text{其它的情况})$$

齿轮的弯曲疲劳极限 σ_{FLim} 是服从对数正态分布的随机变量,由区域图计算 σ_{FLim} 分布参数的公式如下^[1]

$$\mu_{\ln\sigma} = 2.33V_{\ln\sigma} + \ln X \quad (6)$$

X 为在区域图中选择的弯曲疲劳极限,一般取区域图的中点; $\mu_{\ln\sigma}$ 、 $V_{\ln\sigma}$ 分别为 σ_{FLim} 的对数均值和标准差。 $V_{\ln\sigma}$ 的值见文献[1]的表 14-4。

根据齿轮弯曲疲劳极限 σ_{FLim} 和循环基数 N_0 的定义可知, (σ_{FLim}, N_0) 本身就是一组试验值。根据 K_w 的定义式(2)及式(5),把 (σ_{FLim}, N_0) 代入便有

$$K_w = N_0 (\sigma_{FLim} Y_{ST} Y_{\delta relT} Y_{R relT} Y_X)^m \quad (7)$$

对上式二边取对数

$$\ln K_w = \ln N_0 + m [\ln \sigma_{FLim} + \ln Y_{ST} + \ln Y_{\delta relT} + \ln Y_{R relT} + \ln Y_X] \quad (8)$$

式中,齿轮弯曲疲劳极限 S-N 曲线的指数 m 及循环基数 N_0 根据文献[3]图 2.4-35 确定。

在式(8)中, Y_{ST} 、 $Y_{\delta relT}$ 、 $Y_{R relT}$ 和 Y_X 是修正系数,而 N_0 和 m 又是常数,故可以认为 $\ln K_w$ 同 $\ln \sigma_{FLim}$ 的分布类型是一致的,即 K_w 也服从对数正态分布。 K_w 的对数正态分布的均值 μ_{KW} 和标准差 V_{KW} 为

$$\mu_{KW} = \ln N_0 + m [\mu_{\ln\sigma} + \ln \mu_{YST} + \ln \mu_{YR relT} + \ln \mu_{YX}] \quad (9)$$

$$V_{KW} = m (V_{\ln\sigma}^2 + C_{YST}^2 + C_{Y\delta relT}^2 + C_{YR relT}^2 + C_{YX}^2)^{0.5} \quad (10)$$

4 齿轮弯曲疲劳的总疲劳损伤量 D_w

由于齿轮工况多变, 受载情况及工艺水平各异, 并且齿轮轮齿空间有限, 一般较难获得齿轮弯曲应力的应力谱。下面用齿轮弯曲疲劳的载荷谱来计算齿轮的总疲劳损伤量 D_w 。

计算齿轮的计算弯曲应力 σ_F 为^[3]

$$\sigma_F = F_1 K_V K_{F\alpha} K_{F\beta} Y_{Fa} Y_{Fsa} Y_{\epsilon} Y_{\beta} / (b m_n) \quad (11)$$

b, m_n, Y_{ϵ} 及 Y_{β} 这些变量可能的取值区间很小, 因而处理成确定性的变量。其余的 5 个修正变量 $K_V, K_{F\alpha}, K_{F\beta}, Y_{Fa}$ 及 Y_{Fsa} 均处理成正态随机变量, 它们的均值按国标方法计算或查齿轮手册, 变异系数为^[1]

$$C_{K_V} = (\mu_{K_V} - 1) / (3\mu_{K_V}), \quad C_{K_{F\beta}} = 0.05, \quad C_{K_{F\alpha}} = 0.033$$

$$C_{Y_{Fa}} = 0.033, \quad C_{Y_{Fsa}} = 0.04$$

$K_V, K_{F\alpha}, K_{F\beta}, Y_{Fa}$ 及 Y_{Fsa} 与 F_t 是相互独立的随机变量。根据式(3), D_w 的表达式为式(12)。

$$D_w = N [K_V K_{F\alpha} K_{F\beta} Y_{Fa} Y_{Fsa} Y_{\epsilon} Y_{\beta} / (b m_n)]^m \int (F_t)^m g(F_t) dF_t \quad (12)$$

$g(F_t)$ 为弯曲疲劳载荷 F_t 的概率密度函数。

5 齿轮弯曲疲劳可靠度 R 的计算

对具体问题, D_w 表达式中各变量都是已知的, K_w 的分布类型和分布参数也已知, 此时可采用一种通用的计算可靠度的计算方法——等效正态分布法来计算可靠度^[4]。由于 $P(K_w > D_w) = P[\ln K_w > \ln D_w]$, 故极限状态方程见式(13)。

$$\ln X_1 = \ln(K_w), \quad X_2 = K_V, \quad X_3 = K_{F\alpha}, \quad X_4 = K_{F\beta}, \quad X_5 = Y_{Fa}, \quad X_6 = Y_{Fsa}$$

$$A = \ln \{ N [Y_{\epsilon} Y_{\beta} / (b m_n)]^m \int (F_t)^m g(F_t) dF_t \}$$

$$Z = X_1 - A - m [\ln X_2 + \ln X_3 + \ln X_4 + \ln X_5 + \ln X_6] \quad (13)$$

由前面的分析可知 $X_1, \dots, X_6$ 都服从正态分布, 设它们的均值为 μ_i , 标准差为 V_i 。根据等效正态分布法^[4], 求可靠度的迭代方程如下

$$\begin{cases} \cos \alpha_i = -V_i Z'_{xi} / \left(\sum_{j=1}^6 (V_j Z'_{xj})^2 \right)^{0.5} & (i = 1, 2, \dots, 6) \\ X_i^{(2)} = \mu_i + \beta^{(2)} V_i \cos \alpha_i & (i = 1, 2, \dots, 6) \\ Z = X_1^{(2)} - A - m [\ln X_2^{(2)} + \ln X_3^{(2)} + \ln X_4^{(2)} + \ln X_5^{(2)} + \ln X_6^{(2)}] = 0 \end{cases} \quad (14)$$

式中 $Z'_{xi} / P^{(1)}$ 表示极限状态方程(13)对 X_i 的偏导在验算点 $P^{(1)}(X_1^{(1)}, \dots, X_6^{(1)})$ 处取值, $X_i^{(1)}$ 是验算点 $P^{(1)}$ 的坐标值, $X_i^{(2)}$ 是新的验算点 $P^{(2)}$ 的坐标值, $\beta^{(2)}$ 是与验算点 $P^{(2)}$ 对应的可靠指标。迭代方程组(14)收敛条件是

$$|\Delta \beta| < 0.0001 \quad (15)$$

式中 $\Delta \beta$ 表示相邻二个验算点对应的可靠指标之差

当迭代方程(14)收敛时, 可靠度 R 为

$$R = \Phi(\beta) \quad (16)$$

用迭代方程组(14)求可靠度的主要步骤如下:

- (1) 任选初始验算点 $P^{(1)}$ 的坐标值, 但一般选各随机变量的均值作为相应的坐标值。
- (2) 用方程组(14)计算下一个新的验点 $P^{(2)}$ 的坐标值及对应的可靠指标 $\beta^{(2)}$ 。
- (3) 判断收敛条件(15)是否满足, 若不满足则重复(2)-(3)步, 若满足则到(4)步。

(4)由式(16)根据最后一次验算点对应的 $\beta^{(2)}$ 计算可靠度。

6 算 例

某齿轮传动的参数如下: $m_n=4\text{mm}$, $Z_1=14$, $x_1=0.04$, $Z_2=47$, $x_2=-0.04$, $b_1=46\text{mm}$, $b_2=52\text{mm}$, $\beta=8^\circ 34'$, $i=3.6$, $n_1=76.5\text{r/min}$, 额定转矩 $T_1=353\text{N}\cdot\text{m}$ 。小齿轮材料 20MnTiB, 渗碳淬火, HRC=56-62, 大齿轮材料 40Cr, 淬火 HRC=50-56, 精度等级 8-D, 机械油 HJ-50 润滑。其余参数见表 1。试计算该斜齿圆柱齿轮小齿轮弯曲疲劳的可靠度。

解: (1)计算 K_w 的分布参数。根据式(6)、式(9)和式(10), $\mu_{\ln\sigma}$, μ_{KW} 和 V_{KW} 分别为

$$\mu_{\ln\sigma} = 6.3795 \quad \mu_{KW} = 76.839664 \quad V_{KW} = 1.804803$$

(2)计算 D_w 。根据式(12) D_w 为

$$D_w = 3.061126 \times 10^{21} (K_V K_{Fa} K_{F\beta} Y_{Fa} Y_{Sa})^m$$

(3)计算可靠度 R 。根据表 1 有关参数,用迭代方程组(14)进行迭行。根据迭代结果,可靠度 R 为

$$R = \Phi(\beta) = \Phi(4.363335) = 0.9999936$$

表 1 算例各参数

μ_{YST}	2.0	μ_{YRelT}	1.05	N_T	3×10^6	$\mu_{KF\beta}$	1.40	$\mu_{KF\alpha}$	1.45	$\mu_{YS\alpha}$	1.75	N	4.6×10^6
C_{YST}	0.033	C_{YRelT}	0.033	$V_{\ln\sigma}$	0.20	$V_{KF\beta}$	0.07	$V_{KF\alpha}$	0.07485	$V_{YS\alpha}$	0.07	Y_β	0.97
$\mu_{Y\delta RelT}$	1.0	μ_{YX}	1.0	m	8.69565	μ_{KV}	1.04	Y_ϵ	0.77	$\mu_{YF\alpha}$	2.39	F_t	12480
$C_{Y\delta RelT}$	0.03	C_{YX}	0.0	X	370	V_{KV}	0.01352	b	46.0	$V_{YF\alpha}$	0.07887	m_n	4

文献[1]的计算可靠度 0.999997, 本文的计算结果同它非常接近。

按常规弯曲疲劳计算时,安全系数 $S_H=1.73$ 。齿轮手册^[3]表 2.4-2 指出,当 $S_H=1.5$ 时,可靠度 $R>0.9999$ 。本例的计算结果与安全系数计算是相近的,因而本文提供的算法具有相当的计算精度。

7 结束语

本文从一个新的观点出发,把齿轮弯曲疲劳失效处理成随机事件,用齿轮弯曲疲劳累积损伤强度 K_w 和总的疲劳损伤量 D_w 这两个随机变量来描述这个随机事件,提出了计算齿轮弯曲疲劳可靠度的方法。该方法可用来计算单级、多级等幅或随机变幅载荷作用时齿轮的弯曲疲劳的可靠度。本文提供的方法具有相当的计算精度。

参 考 文 献

- 1 卢玉明 机械零件的可靠度设计 北京:高等教育出版社,1989
- 2 乐晓斌,胡宗武,范祖尧 疲劳累积损伤理论的一种新的表达方法 机械强度,1992
- 3 齿轮手册编委会 齿轮手册 北京:机械工业出版社,1990
- 4 卢玉明 通用的可靠度计算方法——等效正态分布法 机械设计,1990

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PERTURBATION TECHNIQUE FOR RELIABILITY ANALYSIS OF A ROTATING DISK WITH UNCERTAIN GEOMETRY

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ABSTRACT Design methods for the aerospace technology are, in general, based on deterministic parameters. In practice there always exist random deviations of parameters, such as material properties and geometry, forces, and boundary conditions. In most situations the random deviations are small, but their overall effect can have a strong impact on behaviours of critical components, particularly in failure and reliability analysis. For this reason a probabilistic finite element method based on second-order perturbation technique is used to analyze the reliability and the random stresses of a rotating disk with uncertain geometry, the elementary perturbation approach to this problem has been studied and formulas for the random stress and the reliability have been deduced. The effect of uncertain geometry on the reliability is considered by a simple numerical example. The accuracy and efficiency of the method is demonstrated by its application to a rotating disk with uncertain geometry. The reliability calculated by this method compares favorably with that obtained from Monte Carlo simulation.

Key words Aircraft engines Perturbation method Structural reliability
Finite element methods

STRESS ANALYSIS OF CHECK RING CONNECTING STRUCTURE BETWEEN COMPOSITE CASE AND NOZZLE

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ABSTRACT The check ring connecting structure between composite case of solid propellant rocket motor and conic fixed body of a nozzle is of great advantage to decrease the mass and stress. In order to analyze this structure an equivalent orthotropic axis-symmetric body is used to represent composite motor case and a FEM program for contact problem with large deformations is developed. The calculation shows that the relative displacement between check ring and nozzle conic fixed body and the angle between two sealing surfaces are very small under the working load. This fact indicates that the check ring connecting structure is safe in application. The relations of the concerned structure parameters to the deflection of the sealing surfaces are discussed. For the sake of comparison it is also found that the structure of submerged fitting increases the contact area magnificently and decrease the angle between two sealing surfaces to 4.02%.

Key words Composite materials Jet nozzles Structural analysis Deformation method

PREDICTION OF GEAR BENDING FATIGUE STRENGTH RELIABILITY

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ABSTRACT A new viewpoint is taken that the fatigue failure of the gear under bending load is treated as a random event, and a method for predicting the gear fatigue strength reliability

ty under bending load is obtained from researching the event. Two new random variables K_w and D_w are proposed to describe the random event of bending fatigue failure of the gear. K_w is the fatigue damage strength which indicates the inherent ability of the gear to resist the fatigue damage under bending load. D_w is the overall fatigue damage quantity which indicates the fatigue damage degree of the gear under bending load in the life span. A cumulative fatigue damage probabilistic model of the gear fatigue strength estimation under bending load is established by means of K_w and D_w . Based on the model, a predicting method is provided for the gear bending fatigue reliability.

Key words Gears Rotational bending fatigue Computational methods Reliability

EXPERIMENTAL INVESTIGATION ON CONVECTIVE BURNING IN A BED OF GRANULAR PROPELLANT BED

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ABSTRACT A semi-closed bomb with 100 MPa pressure limits was designed to investigate the ignition transient, flame spreading, and pressure wave propagation in a packed bed of granular propellant charge. The influences of parameters such as propellant properties, porosity, and blow-out diaphragm thickness on the convective burning characteristics were examined. The 1-D pressure wave propagation was observed in the experiment. The dynamic impaction of the bed exists during the flame spreading process, which is a main factor to cause the pressure wave. The 1-D combustion wave can not be formed in the bed with low loading density.

Key words Propellant combustion Pressure waves Internal ballistic tests
Combustion stability

EXPERIMENTAL INVESTIGATION ON FULL COVERAGE DISCRETE HOLE FILM COOLING

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ABSTRACT A heat shield with full coverage discrete hole film cooling has practical application in some afterburners. Effects of geometric parameters and flow parameters of the heat shield on effectiveness of film cooling were investigated experimentally. A convective heat transfer coefficient of the film cooling was defined as $h_g = q_c / (T_g - T_w)$, where q_c is heat flux obtained by solving heat balance equation for the tested plate with discrete holes, and T_g and T_w are gas temperature and tested plate wall temperature, respectively. A semiempirical expression for calculating heat transfer coefficient is provided for a given diameter of the discrete holes and their distribution. It might be useful to calculating heat shield temperature.

Key words Air film cooling blades Cooling Experiment Combustion chambers

MULTI-HOLE COOLING TECHNOLOGY ANALYSES

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ABSTRACT This paper is to analyse the advantage of the multi-hole cooling technology. The calculation showed that the combustor liner temperature with multi-hole cooling technology would be 150K to 200K less than the wall temperature with film cooling technology, and the tem